Captivate
EDUCATION

Specialist Mathematics

Question and Answer Book

Practice examination 1 – Examination 1 – 2027

Student name _____

Teacher _____

- Reading time is 15 minutes
- Writing time is 1 hour

Materials supplied

- Question and Answer Book
- Formula Sheet

Students are not permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

10 questions (40 marks)

Practice material in the layout of the 2025 VCAA paper – generated 2027. Not a VCAA examination.

SAMPLE

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- Unless otherwise specified, an exact answer is required for each question.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.
- Take the acceleration due to gravity to have magnitude g m s−2, where g = 9.8

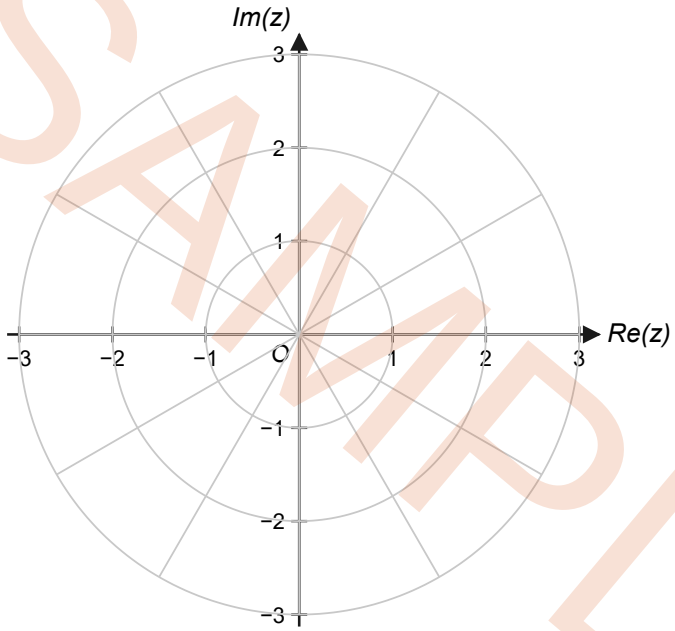
SAMPLE

Question 1 (4 marks)

Let $P(z) = z^3 - z^2 + 3z + 5$, where $z \in \mathbb{C}$.

a. Given that $z = 1 - 2i$ is a solution of $P(z) = 0$, find the other two solutions. 2 marks

b. Plot and label the three solutions of $P(z) = 0$ on the Argand diagram below. 1 mark



c. The points that represent the three solutions of $P(z) = 0$ are the vertices of a triangle in the complex plane. Find the area of this triangle. 1 mark

SAMPLE

Question 2 (3 marks)

Consider the curve with equation $2x^2 - xy + y^2 = 7$.

Find the coordinates of the points on the curve at which the tangent to the curve is horizontal. 3 marks

Question 3 (4 marks)

Let $f: (-2, 2) \to \mathbb{R}, f(x) = x\sin^{-1}\left(\frac{x}{2}\right) + \sqrt{4 - x^2}$.

a. Show that $f'(x) = \sin^{-1}\left(\frac{x}{2}\right)$. 2 marks

b. Hence, find $\int_0^1 \sin^{-1}\left(\frac{x}{2}\right) dx$. 2 marks

SAMPLE

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Marking scheme – teacher copy

Practice examination 1 – Examination 1 – 2027

Not for distribution to students.

Section	Questions	Marks available	Marks awarded
Paper	10	40	
Total		**40**	

Question 1a. (2 marks)

Given that $z = 1 - 2i$ is a solution of $P(z) = 0$, find the other two solutions.

To be awarded full marks, students were required to:

- **M1** conjugate 1 + 2*i* stated, or the real quadratic factor $z^2 - 2z + 5$ found (can be inferred)
- **A1** both 1 + 2*i* and −1

Does not score: applying the conjugate root theorem without noting that the coefficients are real is not penalised here, but a response giving only 1 + 2*i* earns M1 only; −1 + 2*i* (a sign slip in the conjugate); dividing by *z* − (1 − 2*i*) alone and leaving a quadratic with complex coefficients unsolved.

Mark allocation: M1 + A1 = 2 marks

Sample full-mark response

P has real coefficients, so by the conjugate root theorem $z = 1 + 2i$ is also a solution. M1

$(z - (1 - 2i))(z - (1 + 2i)) = (z - 1)^2 + 4 = z^2 - 2z + 5$ is a factor of *P*(*z*).

Comparing coefficients: $z^3 - z^2 + 3z + 5 = (z^2 - 2z + 5)(z + 1)$, so the third solution is $z = -1$.

The other two solutions are $z = 1 + 2i$ and $z = -1$. A1

SAMPLE

Question 1b. (1 mark)

Plot and label the three solutions of $P(z) = 0$ *on the Argand diagram below.*

To be awarded full marks, students were required to:

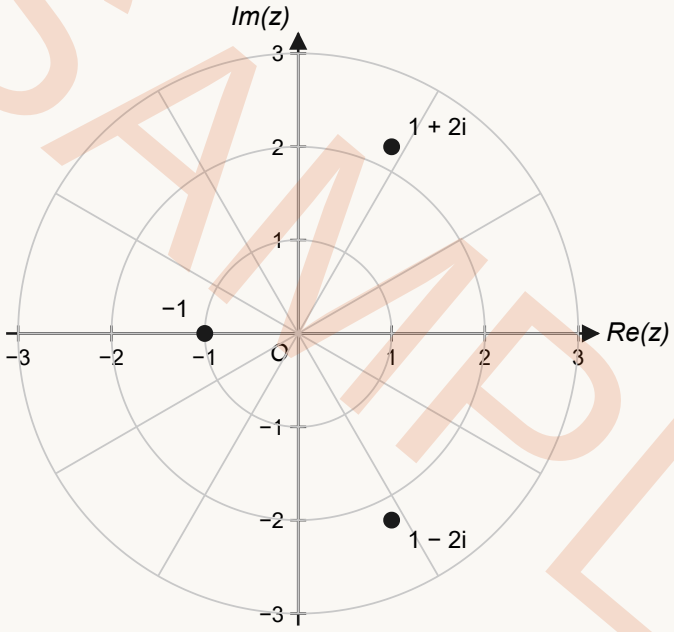
- **A1** all three points plotted correctly and labelled; consequential (H1) on the solutions found in part a

Does not score: 1 + 2*i* plotted at (2, 1) (real and imaginary parts interchanged); −1 plotted on the imaginary axis; points not labelled.

Mark allocation: A1 = 1 mark

Sample full-mark response

Stimulus specification (expected plot, on the printed Argand diagram with rings of radius 1, 2, 3): points at (1, 2), (1, −2) and (−1, 0), each labelled with its value.



Points 1 + 2*i* at (1, 2), 1 − 2*i* at (1, −2) and −1 at (−1, 0), each labelled. A1

SAMPLE

Question 1c. (1 mark)

The points that represent the three solutions of $P(z) = 0$ *are the vertices of a triangle in the complex plane. Find the area of this triangle.*

To be awarded full marks, students were required to:

- **A1** 4; consequential (H1) on part a when the solutions are a conjugate pair and a real number

Does not score: 8 (the base times the height without the factor ½); using $|1 + 2i| = \sqrt{5}$ as a side length of a right-angled triangle.

Mark allocation: A1 = 1 mark

Sample full-mark response

The vertices are (1, 2), (1, −2) and (−1, 0). The side joining $1 \pm 2i$ has length 4 and the perpendicular distance from −1 to the line $\text{Re}(z) = 1$ is 2.

Area $= \frac{1}{2} \times 4 \times 2 = 4$ square units A1

SAMPLE