Captivate
EDUCATION

Mathematical Methods

Question and Answer Book

Practice examination 1 – Examination 1 – 2027

Student name _____

Teacher _____

- Reading time is 15 minutes
- Writing time is 1 hour

Materials supplied

- Question and Answer Book
- Formula Sheet

Students are not permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

9 questions (40 marks)

Practice material in the layout of the 2025 VCAA paper – generated 2027. Not a VCAA examination.

SAMPLE

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Question 1 (3 marks)

a. Let $y = x \log_e(3x)$. Find $\frac{dy}{dx}$. 1 mark

b. Let $f(x) = \frac{\sin(2x)}{x}$. Find $f'(\frac{\pi}{4})$. 2 marks

Question 2 (2 marks)

Let *f*: $R \rightarrow R$ be a function such that $f'(x) = \cos(2x) - e^{-x}$ and $f(0) = 2$.

Find *f*(*x*). 2 marks

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Question 3 (4 marks)

a. Solve $e^{2x} + 2e^{x} - 15 = 0$ for x. 2 marks

b. Solve $\log_2(x + 2) + \log_2(x - 1) = 2$ for x. 2 marks

SAMPLE

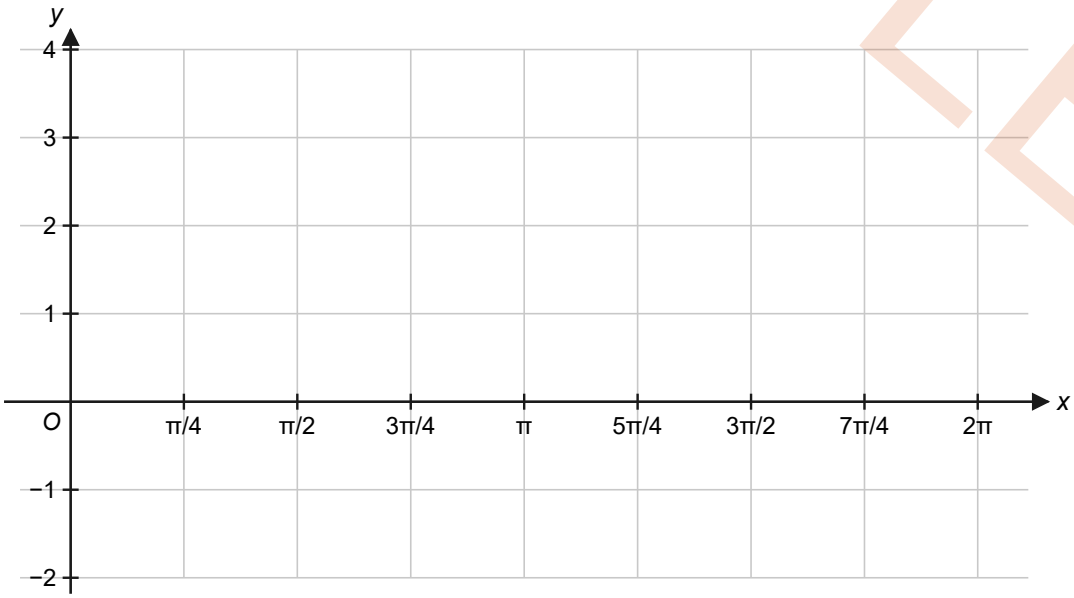
Question 4 (6 marks)

Let $f: [0, 2\pi] \to R,\ f(x) = 1 - 2\sin(2x)$.

a. State the range of *f*. 1 mark

b. Solve $f(x) = 0$ for *x*. 3 marks

c. Sketch the graph of $y = f(x)$ on the axes below. Label the endpoints and the stationary points with their coordinates. 2 marks



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Marking scheme – teacher copy

Practice examination 1 – Examination 1 – 2027

Not for distribution to students.

Section	Questions	Marks available	Marks awarded
Paper	9	40	
Total		**40**	

Question 1a. (1 mark)

Let $y = x \log_e(3x)$. *Find* $\frac{dy}{dx}$.

To be awarded full marks, students were required to:

$dy/dx = \log_e(3x) + 1$

Mark allocation: A – accept $\log_e(3) + \log_e(x) + 1$ or any equivalent form; the second term must be simplified to 1

Does not score: 3/(3x) left unsimplified inside an otherwise correct answer is accepted only if the final line is simplified; $\log_e(3x)$ alone (second term missing); 1/(3x) used as the derivative of $\log_e(3x)$.

Sample full-mark response

Product rule: $u = x$, $v = \log_e(3x)$, $dv/dx = 3/(3x) = 1/x$

$dy/dx = \log_e(3x) + x \times 1/x = \log_e(3x) + 1$

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Question 1b. (2 marks)

Let $f(x) = \frac{\sin(2x)}{x}$. *Find* $f'(\frac{\pi}{4})$.

To be awarded full marks, students were required to:

$f'(\pi/4) = -16/\pi^2$

Mark allocation: M – correct quotient (or product) rule derivative, including the chain-rule factor 2
A – $-16/\pi^2$; accept equivalent exact forms
Does not score: the numerator in the wrong order (giving $+16/\pi^2$); cos(2x) differentiated without the factor 2; leaving cos(π/2) and sin(π/2) unevaluated.

Sample full-mark response

> Quotient rule: $f'(x) = (2x\cos(2x) - \sin(2x)) / x^2$
>
> $f'(\pi/4) = (2 \times \pi/4 \times \cos(\pi/2) - \sin(\pi/2)) / (\pi/4)^2 = (0 - 1) / (\pi^2/16) = -16/\pi^2$

Question 2. (2 marks)

Find f(x).

To be awarded full marks, students were required to:

$f(x) = \frac{1}{2}\sin(2x) + e^{-x} + 1$

Mark allocation: M – anti-derivative with both coefficients correct (½ and +1 on e^{-x}) and a constant of integration
A – $c = 1$ used and the rule stated as $f(x) = \ldots$; any equivalent form
Does not score: $-e^{-x}$ as the anti-derivative of $-e^{-x}$ (sign error); 2 sin(2x) for the first term; no constant of integration — the 2021 Examination 1 report notes the constant is required.

Sample full-mark response

> $f(x) = \int (\cos(2x) - e^{-x})\,dx = \frac{1}{2}\sin(2x) + e^{-x} + c$
>
> $f(0) = 0 + 1 + c = 2$, so $c = 1$
>
> $f(x) = \frac{1}{2}\sin(2x) + e^{-x} + 1$

SAMPLE

Question 3a. (2 marks)

Solve $e^{2x} + 2e^{x} - 15 = 0$ *for x.*

To be awarded full marks, students were required to:

$x = \log_e(3)$

Mark allocation: M – the equation treated as a quadratic in e^x and factorised (or solved)
A – $x = \log_e(3)$ only, with $e^x = -5$ rejected
Does not score: including $\log_e(-5)$ as a solution — the 2023 and 2025 Examination 1 reports both record a failure to observe $e^x > 0$; giving $x = 3$.

Sample full-mark response

> Let $u = e^x$: $u^2 + 2u - 15 = 0 \Rightarrow (u + 5)(u - 3) = 0$
>
> $e^x = -5$ has no solution since $e^x > 0$; $e^x = 3 \Rightarrow x = \log_e(3)$

Question 3b. (2 marks)

Solve $\log_2(x + 2) + \log_2(x - 1) = 2$ *for x.*

To be awarded full marks, students were required to:

$x = 2$

Mark allocation: M – log laws used correctly to form $(x + 2)(x - 1) = 2^2$
A – $x = 2$ only; -3 rejected
Does not score: keeping $x = -3$ (outside the domain of $\log_2(x - 1)$); the 2024 Examination 1 report notes students who did not reject invalid solutions); writing $(x + 2)(x - 1) = 2$.

Sample full-mark response

> $\log_2((x + 2)(x - 1)) = 2 \Rightarrow (x + 2)(x - 1) = 4$
>
> $x^2 + x - 6 = 0 \Rightarrow (x + 3)(x - 2) = 0 \Rightarrow x = -3$ or $x = 2$
>
> The equation requires $x > 1$, so $x = -3$ is rejected: $x = 2$ (check: $\log_2(4) + \log_2(1) = 2 + 0 = 2$)

SAMPLE